

Coursework: Exposition of an Open Problem

Commutative Algebra · Worth 10% of the module mark

Find an open problem in commutative algebra that can be stated using the material of this course, and write an exposition of it. There are two submissions:

1. a **proposal** of at most one page, due **20 November**;
2. an **exposition** of at most five pages, due **10 December**.

I will approve your proposal or ask you to change it within a week, and when I approve it I will assign you a specific case to work out in your exposition.

1. The proposal (at most one page)

- (a) **Statement.** State the problem precisely, using definitions from the course. If there are several versions, say which one you mean.
- (b) **Source.** Give a reference where the problem was posed.
- (c) **Evidence that it is open.** Give a reference stating that the problem is open, and say briefly how you checked that it has not been solved since.
- (d) **Fit with the course.** List the notions from the course needed to state the problem, and any notions not in the course that you will have to define.

The problem must be a well-posed question whose answer is not known ("Is every X a Y ?"), not a vague research direction.

2. The exposition (at most five pages)

Written in $\text{\LaTeX}$, 11pt, standard margins. References and an optional appendix of computer code do not count towards the limit. Your exposition should contain:

1. **The problem**, stated precisely, with every notion not in the course notes defined.
2. **Context**: why the problem is natural and how it relates to results of the course.
3. **A worked example**: the case assigned to you, worked out in full.
4. **What is known**, including a proof or careful sketch of at least one known case using only course material.
5. **Status**: why you believe the problem is still open as of your submission date, with sources.
6. **References**, cited where they are used.

Precision matters more than breadth: a short, correct account of one special case is better than a long list of results you cannot explain.

3. Sources and AI tools

- You may use AI tools for searching the literature and for help with $\text{\LaTeX}$, provided you declare this at the end of your exposition.
- You are responsible for every statement and reference. AI tools can misreport whether a problem is open. Check every reference yourself; a reference that does not exist or does not say what you claim is a serious error.
- Copying or closely paraphrasing any source, including AI output, without attribution is plagiarism.

Note. This is a preliminary version of the coursework instructions. If you have any comments or concerns, please feel free to e-mail me or come to office hours to chat.