**Homework 8:**

1) (encore) If $\sigma : \Delta_n \to X$ is a simplex, define $\overline{\sigma} : \Delta_n \to X$ by

$$\overline{\sigma}(t_0, \dots, t_n) := \sigma(t_n, \dots, t_0).$$

Define a chain map $T : C_n(X) \to C_n(X)$ by $T(\sigma) := (-1)^{n(n+1)/2}\overline{\sigma}$. Show that there exists a chain homotopy from $T$ to the identity. (Hint: You don't need to construct the chain homotopy explicitly.)

2) Show that $H_n(S^n, S^n - \{q\}) \cong H_n(S^n)$ for any point $q \in S^n$. More generally, when $M$ is a manifold of dimension $n$, and $q \in M$, show that $H_n(M, M - \{q\}) \cong \mathbb{Z}$ $\cancel{H_n(M)}$.

3) Hatcher, page 155, pb. 4.

4) Hatcher, page 155, pb. 8.

5) Compute $H_*(\mathbb{C}P^n)$, $H_*(S^n \times S^n)$ and $H_*(\mathbb{R}P^n)$ for all $n$.

6) Hatcher, page 156, pb. 14.

7) Hatcher, page 157, pb. 17.

8) Hatcher, page 157, pb. 28.

9) Show taht $S^2 \vee S^4$ and $\mathbb{C}P^2$ have isomorphic homology groups. (We will see later that these homotopy types can be distinguished by cohomology rings).