

Homework 7:

Below, if unspecified, take the coefficient ring R to be \mathbb{Z} .

- 1) Let $A \subset X$. Prove that $H_k(X \cup CA) \cong H_k(X, A)$ for $k > 0$ by using long exact sequence of the pair $(X \cup CA, CA)$ and excision theorem.
- 2) Note that if A is a CW subcomplex of a CW complex X that is contractible, the quotient map $X \rightarrow X/A$ is a homotopy equivalence (Hatcher Prop. 0.17). Now, in the more general setting, suppose A is a CW subcomplex of a CW complex X . Using the fact that CA can be made into a CW-subcomplex of $X \cup CA$, prove that $H_*(X/A) \cong H_*(X, A)$.
- 3) Hatcher page 132, pb. 26
- 4) Hatcher page 132, pb. 14
- 5) Hatcher page 132, pb. 15
- 6) Hatcher page 132, pb. 17
- 7) Hatcher page 132, pb. 18
- 8) Hatcher page 132, pb. 22.
- 9) Use Mayer-Vietoris sequence to give another computation of $H_*(S^n)$ for all n .
- 10) (Optional) We have discussed the construction of the boundary homomorphism, that allows one to prove that a short exact sequence of *chain complexes* over a ring R induce a long exact sequence of *R-modules*. In homological algebra, this is known as the Snake lemma. Watch it explained in the first scene of Claudia Weill's film *It is My Turn* (1980), starring Jill Clayburgh and Michael Douglas.